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Analytic Representations of SL_2 over a $\mathfrak{p}$-Adic Number Field, III

Yasuo Morita

§ 0. Introduction

0-1. In our former paper [12], we have constructed a p-adic analogue of the holomorphic discrete series of $SL_2(\boldsymbol{R})$ which is related to the theory of p-adic Schottky groups of D. Mumford. We have also constructed a p-adic analogue of the non-unitary principal series in [11], and have studied the relation between our discrete series and our principal series. The main purpose of this paper is to study the irreducibilities and the equivalences of our principal series.

Let $\boldsymbol{Q}_p$ be the p-adic number field, let L be a finite extension of $\boldsymbol{Q}_p$, and let k be a field containing L. We assume (i) the p-adic valuation of $\boldsymbol{Q}_p$ can be extended to a valuation $|\ \ |$ of k, and (ii) k is maximally complete with respect to $|\ \ |$ (cf. § 1 for a definition). These conditions are satisfied if k is a finite extension of L. Let L^* and k^* be the multiplicative groups of L and k, respectively, and let χ: $L^* \to k^*$ be a homomorphism which can be expressed as $\chi(z) = \exp\{\alpha(\chi) \log(z)\}$ for some $\alpha(\chi) \in k$ if z is sufficiently close to 1. Hence χ is a locally analytic character of L^* with values in k^*.

Let G denote the group $SL_2(L)$, and let P be the subgroup of G of all lower triangular matrices. We define a one-dimensional representation χ of P by

$$P \ni \begin{pmatrix} a & 0 \\ c & d \end{pmatrix} \longmapsto \chi(a) \in k^*,$$

and construct the induced representation Ind (P, G, χ) of G in the category of k-valued locally analytic functions (cf. § 1 and § 2 for the exact definition). Further we realize this representation of G on a space D_χ of k-valued locally analytic functions on L in a natural manner. Then it is not difficult to find all closed G-invariant subspaces of D_χ. For simplicity, we assume that $\alpha(\chi)$ is not a non-negative integer. Then our main result in this case can be stated as the following:

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