Advanced Studies in Pure Mathematics 7, 1985
Automorphic Forms and Number Theory
pp. 113–148

# Group Cohomology and Hecke Operators 2
# Hilbert Modular Surface Case

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In the previous report [27], the authors developed the functorial behavior of Hecke operators operating on group cohomologies, and applied them to arithmetic Fuchsian groups to prove some congruence relations between eigenvalues of Hecke operators. There the authors promised to present similar congruence relations for Hilbert modular groups by the same principle. The author of the present report partly fulfills the promise. Namely, Theorem (3.4) in the last page of this report is a direct analogue for Hilbert-modular-surface case of Theorem (2.2.3) of the previous report [27].

The author would like to thank the following people:

(1) Walter Parry and Chih-Han Sah were co-authors of the previous report. Occasional conversations with them were very helpful to prepare this report, especially in 1.6 and cohomology of $\Gamma^{\mathrm{ab}}$.

(2) Salman Abdulali, Min Ho Lee, Troels Petersen and Monica Petri were note-takers and helpful critics of my lectures on this subject. Based on their notes, this report was made.

## § 1. Notations and known facts

**1.1.** Standard notations of $\boldsymbol{Z}$, $\boldsymbol{Q}$, $\boldsymbol{R}$, $\boldsymbol{C}$ are used for the ring of integers, fields of rationals, reals, and complex numbers. In general, the notation $K$ denotes a field of characteristic zero, usually $K=\boldsymbol{Q}$, $\boldsymbol{R}$ or $\boldsymbol{C}$. The finite field with $q$ elements is denoted by $\boldsymbol{F}_q$. For an integer $n$, the cyclic group of order $n$ is denoted by $\boldsymbol{Z}_n$ or by $\boldsymbol{Z}/n\boldsymbol{Z}$; if $n$ is a prime number $\ell$ it is also denoted by $\boldsymbol{F}_\ell$. In general, for a module $\mathscr{M}$ and an integer $n$, the cokernel $\mathscr{M}/n\mathscr{M}$ of the $n$-multiplication $x\to nx$ is denoted by $\mathscr{M}_n$, and the kernel is denoted by ${}_n\mathscr{M}=\{x\in\mathscr{M}; nx=0\}$.

In this note, as a module $\mathscr{M}$ we consider $\mathscr{M}=\boldsymbol{C}$, $\boldsymbol{R}$, $\boldsymbol{Q}$, $\boldsymbol{Z}$, $\boldsymbol{Z}_n$, in particular $\boldsymbol{F}_\ell$, $\boldsymbol{C}/\boldsymbol{Z}$, $\boldsymbol{R}/\boldsymbol{Z}$, $\boldsymbol{Q}/\boldsymbol{Z}$, or a finite direct sum of them; so $\mathscr{M}_n$ and ${}_n\mathscr{M}$ are always finite modules.

For a group $G$, $G/[G, G]$ is denoted by $G^{\mathrm{ab}}$; $[G, G]$ is denoted by $G^{(1)}$,

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Received June 21, 1984.