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Moonshine for $PSL_2(F_7)$

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0. In [1], Conway and Norton assigned a Thompson series of the form

$$q^{-1}+\sum_{n=1}^{\infty} H_n(m)q^n, \quad q=e^{2\pi i z}$$

to each element m of the Fischer-Griess group F_1 where H_n are characters of F_1, and they conjectured among others that Thompson series are generators of the modular function fields of genus zero for some modular groups which contain $\Gamma_0(N)$ for some N. In [6], Queen studied moonshine for other simple groups, for example, Thompson's group F_3.

In this paper, we consider these phenomena for $PSL_2(F_7)$ and its relation to Conway-Norton's monstrous moonshine.

Let $G=PSL_2(F_7)$. G acts on $F_7 \cup \{\infty\}$ as linear fractional transformations, so G can be considered as the subgroup of S_8. Then, each element of G is written by products of cycles and these are of the following forms:

$$1^8,\ 1\cdot 7,\ 1^2\cdot 3^2,\ 2^4,\ 4^2.$$

For each product of cycles of length n_i, $m=(n_1)(n_2)\cdots(n_s)$, $n_1\geq\cdots\geq n_s\geq 1$ $\sum_{i=1}^{s} n_i=8$, in G, we associate following modular forms:

$$\eta_{1,m}(z)=\prod_{i=1}^{s}\eta(3n_i z),$$

$$\eta_{2,m}(z)=\prod_{i=1}^{s}\eta(n_i z)^3,$$

where $\eta(z)$ is the Dedekind η-function. Then $\eta_{1,m}(z)$ (resp. $\eta_{2,m}(z)$) is a cusp form of weight $s/2$ (resp. $3s/2$) on $\Gamma_0(9n_1n_s)$ (resp. $\Gamma_0(n_1n_s)$) with some character and is known to be a common eigenfunction of all Hecke operators (cf. [4]).

We shall prove

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