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On Relations of Dimensions of Automorphic Forms of $Sp(2, \boldsymbol{R})$ and Its Compact Twist $Sp(2)$ (II)

Ki-ichiro Hashimoto and Tomoyoshi Ibukiyama*)

In this paper, we show some good global dimensional relations between automorphic forms of $Sp(2, \boldsymbol{R})$ (matrix size four) and its compact twist $Sp(2)$. One of the authors has already shown such relations when the p-adic completions (for a fixed prime p) of the discrete subgroups in question are maximal compact (See [24]). In this paper, we treat discrete subgroups whose p-adic completions are minimal parahoric. Our aim is a generalization of Eichler-Jacquet-Langlands correspondence between SL_2 and $SU(2)$ to the symplectic case of higher degree. Such correspondence should be proved by comparison of the traces of all the Hecke operators. Our results mean that there exist relations of traces at least for $T(1)$ for some explicitly defined discrete subgroups of $Sp(2, \boldsymbol{R})$ and $Sp(2)$ (§ 2 Main Theorem I). Besides, they give meaningful examples for Langlands philosophy on stable conjugacy classes (§ 2 Main Theorem II). Roughly speaking, such comparison is divided into character relations at infinite places (which are more or less known) and arithmetics at finite places. Our point is to execute the comparison of the *arithmetical* part explicitly. It seems that our Theorems are the first global results on such relations except for GL_n (cf. also [24]). In Section 1, after a brief introduction, we give a precise formulation on our problems between $Sp(n, \boldsymbol{R})$ and $Sp(n)$ for general n, e.g. on how to choose discrete subgroups explicitly. For automorphic forms with respect to these explicitly chosen discrete subgroups, we propose there two conjectures (which were first given in [21], [23]): coincidence of dimensions and existence of an isomorphism between new forms as Hecke algebra modules. For $n=1$, these are nothing but the *theorems* by Eichler [10], [11], and the above conjectures are a natural generalization of his results. Langlands [34] has given a quite general philosophy on correspondence of automorphic forms of any reductive algebraic groups, but we understand that his philosophy does not give very detailed formulation at present for such typical and explicit cases as

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