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On Relations of Dimensions of Automorphic Forms of $Sp(2, \mathbf{R})$ and Its Compact Twist $Sp(2)$ (I)

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Let p be a fixed prime. In the previous paper [9], we have given some examples and conjectures on correspondence between automorphic forms of $Sp(2, \mathbf{R})$ (size four) and $Sp(2)=\{g \in \mathbf{H};\ g\,{}^t\bar{g}=1_2\}$ ($\mathbf{H}$: Hamilton quaternions) which preserves L-functions, where the p-adic closures of the discrete subgroups (to which automorphic forms belong) are minimal parahoric. This was an attempt to a generalization of Eichler's correspondence between $SL_2(\mathbf{R})$ and $SU(2)$. Ihara raised such a problem for symplectic groups and Langlands [15] has given a quite general philosophy on correspondence of automorphic forms of any reductive groups (functoriality with respect to L-groups). In this paper, we give good global dimensional relations of automorphic forms of $Sp(2, \mathbf{R})$ and $Sp(2)$, when the p-adic closures of discrete subgroups in question are maximal compact. (As for similar results for other groups, see [8].) More precisely, put

$$K(p)=Sp(2, \mathbf{Q})\cap \gamma M_4(\mathbf{Z})\gamma^{-1}$$
$$=Sp(2, \mathbf{Q})\cap\begin{pmatrix} * & * & */p & * \\ p* & * & * & * \\ p* & p* & * & p* \\ p* & * & * & * \end{pmatrix},\quad \text{where } \gamma=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

and $*$'s run through all integers. For any $\Gamma\subset Sp(2, \mathbf{R})$, denote by $A_k(\Gamma)$ (resp. $S_k(\Gamma)$) the space of automorphic (resp. cusp) forms belonging to Γ. We shall calculate the dimension of $S_k(K(p))$ for all primes p (Theorem 4 in § 4). By comparing these with those of certain automorphic forms (i.e., certain spherical functions) of $Sp(2)$, we shall show certain interesting relations of dimensions (Theorem 1 below). Some philosophical aspects of relations of orbital integrals have been explained in Langlands [16]. But except for the case of GL_n, or the product of its copies, as far as I know, this is the first global result concerning on the comparison of

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