

The Restricted Root System of a Semisimple Symmetric Pair

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§ 0. Introduction

Let \mathfrak{g} be a real semisimple Lie algebra and let σ be an involutive linear automorphism of \mathfrak{g} . If $\mathfrak{h} = \{X \in \mathfrak{g}; \sigma X = X\}$ and $\mathfrak{q} = \{X \in \mathfrak{g}; \sigma X = -X\}$, we obtain a direct sum decomposition $\mathfrak{g} = \mathfrak{h} + \mathfrak{q}$. The pair $(\mathfrak{g}, \mathfrak{h})$ is called a (semisimple) symmetric pair. A classification of such pairs was accomplished by M. Berger [Be]. Then it is important to study the fine structure of a symmetric pair. Among other things, the restricted root system of a symmetric pair is to be determined. One of the purpose of this paper is this. Needless to say, the results of this paper will play a basic role in the study of Fourier analysis on a semisimple symmetric space. This will be treated in the subsequent papers.

This paper deals with the study on the basic structure of a symmetric pair. The main part of this paper is the contents in Section 1-Section 6 and the results of Section 7, Section 8 are preparations of the subsequent papers.

We explain the contents shortly. In Section 1, after giving the defi-