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Boundary Value Problems for Systems of Linear Partial Differential Equations with Regular Singularities

Toshio Oshima

A concept of systems of linear partial differential equations with regular singularities and their boundary value problems were introduced by [K–O]. A typical example is the Laplacian $\Delta=(1-|z|^2)^2\partial^2/\partial z\partial\bar{z}$ on the unit disc in the complex plane $\boldsymbol{C}$, which has regular singularity along the boundary of the disc. In this case S. Helgason proved that any eigenfunction of Δ can be obtained by the Poisson integral of a hyperfunction on the boundary. The inverse correspondence is given by the map of taking the boundary value of the solution, which was defined in [K–O]. In general any simultaneous eigenfunction of the invariant differential operators on a Riemannian symmetric space of the non-compact type can be given by the Poisson integral of a hyperfunction on a boundary of the symmetric space. The main purpose of [K–O] was to prove this statement and in fact it was solved in [K–K–].

When we consider a realization of a Riemannian (or semisimple) symmetric space in a nice compact manifold (cf. [O 2] and [O–S]), the invariant differential operator has regular singularities along the boundaries. Hence for a deeper analysis on a symmetric space, we need a deeper study on systems of differential equations with regular singularities. This is a main motivation to write this paper and several applications of this paper to this subject will appear in subsequent papers. One of them will be found in [MaO].

We will mention some differences between [K–O] and this paper. In this paper we discuss a system of differential equations which has not necessarily one unknown function but finitely many. This enables us to study a system of differential equations defined in a vector bundle over a symmetric space. Moreover in [K–O] we only consider a system of differential equations whose number equals just the codimension of the boundary. But here we remove this restriction and we can consider more equations that the solution satisfies. Then the boundary value of the solution may satisfy some equations. These induced equations will be

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