Advanced Studies in Pure Mathematics 4, 1984
Group Representations and Systems of Differential Equations
pp. 291–330

Geometric Approach to the Completely Integrable Hamiltonian Systems Attached to the Root Systems with Signature

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Introduction

Because of their symmetry, completely integrable Hamiltonian systems are intimately related to the geometry of Lie groups and homogeneous spaces. Conversely it is probable to find new completely integrable Hamiltonian systems among the Hamiltonian systems naturally constructed in connection with Lie groups and homogeneous spaces. From the view point described above, we shall consider in this article the Hamiltonian systems attached to certain root systems with signature. In more detail, we shall treat the following ones. Throughout the paper we retain the following notations. Let n be an integer such that $n \geqq 2$, and let m be an integer satisfying $1 \leqq m \leqq n$. For a notational convenience we write

$$\sum_{(1)} = \sum_{1 \leqq j<k \leqq m,\ m<j<k \leqq n}, \qquad \sum_{(2)} = \sum_{1 \leqq j \leqq m,\ m<k \leqq n}$$

and

$$\mathrm{sh}\,(x) = \sinh\,(x), \qquad \mathrm{ch}\,(x) = \cosh\,(x)$$

and moreover for $q=(q_1, \cdots, q_n) \in \boldsymbol{R}^n$

$$q_{jk} = q_j - q_k, \qquad \hat{q}_{jk} = q_j + q_k.$$

(I) The Hamiltonian system attached to the root system with signature (A_{n-1}, ε_m).

This is the Hamiltonian system on the phase space $D_{(A_{n-1}, \varepsilon_m)} \times \boldsymbol{R}^n$ with the Hamiltonian $H_{(A_{n-1}, \varepsilon_m)}$ where

$$(0.1) \qquad D_{(A_{n-1}, \varepsilon_m)} = \{q=(q_1, \cdots, q_n) \in \boldsymbol{R}^n ; q_1 > \cdots > q_m, q_{m+1} > \cdots > q_n\}$$

and

Received August 10, 1983.