Advanced Studies in Pure Mathematics 4, 1984
Group Representations and Systems of Differential Equations
pp. 267–289

Microlocal Analysis of Theta Functions

Mikio Sato, Masaki Kashiwara and Takahiro Kawai

§ 0. Introduction

The purpose of this report is to show how the theta-zerovalue and some related functions can be controlled microlocally. To be specific, we first prove a result on the constructibility (in particular, finiteness) of the complex of $\Gamma_Z(\varphi_*\mathcal{O})$-solution sheaves for a class of microdifferential equations, which we call $\boldsymbol{R}$-holonomic. (Theorem 1.2 and Theorem 1.3. See (1.8) for the precise definition of $\Gamma_Z(\varphi_*\mathcal{O})$-solutions.) The constructibility of microfunction solution sheaves will then follow from that of $\Gamma_Z(\varphi_*\mathcal{O})$-solution sheaves. (Theorem 1.4.) This finiteness result generalizes the result for holonomic systems ([6]), giving us the hope that $\boldsymbol{R}$-holonomic complexes will be effectively used in application. Our expectation is augmented by the validity of the "Reconstruction Theorem" for $\boldsymbol{R}$-holonomic complexes. (Theorem 1.5.) Furthermore it is really confirmed by the fact that the finiteness theorem for theta-zerovalues, or Jacobi functions, which was announced in Sato [12], follows from the general result for $\boldsymbol{R}$-holonomic complexes. (Theorem 2.8.) Note that the finiteness theorem for theta-zerovalues is closely tied up with their automorphic property. (See § 2.)

We refer the reader to Sato-Kashiwara-Kawai [13], particularly its introduction, for the background of the finiteness theorem given in this report. At the same time, it is worth while noting the following two distinctions between the presentation of [13] and that of this article:

First, in this article we do concentrate our attention on the theta-zerovalues, not the theta functions. The discussion on theta functions can be done in exactly the same manner as in [13], once we get the finiteness theorem (Theorem 2.8) for theta-zerovalues. So, we do not repeat it here. Note, however, that the way of the reasoning in [13] is quite different from the reasoning given here in that the former one first shows some finiteness theorem for theta functions and then deduces from it the finiteness theorem for the theta-zerovalues. Apparentlysuc h an approach is cumbersome in discussing microfunction solutions as we do in this

Received July 19, 1983.