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Infra-red Finiteness in Quantum Electro-Dynamics

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In this talk we want to report some mathematical aspects of a recent solution of the infra-red catastrophe in quantum electro-dynamics. See Stapp [1], [2] for the physical implications and general ideas of this solution. Some mathematical claims made in [1] were verified in Kawai-Stapp [3], upon which we report here. We hope this report will introduce more mathematicians to the infra-red divergence problem, which is becoming increasingly important and intriguing, particularly in connection with the development of *QCD*.

A principal result in [1] is that the coordinate space Feynman function $F^D(x)$ can be separated into two factors the first of which is a unitary operator in photon space representing the classical electro-magnetic contribution to the amplitude, and the second of which is a residual factor representing the quantum fluctuation about the classical contribution.

The main objectives of [3] were to verify:

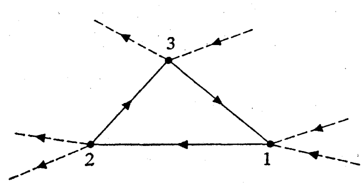
(i) the residual factor is free of infra-red divergences,

and

(ii) the dominant part of the singularity of the residual factor on the positive-α Landau surface $L^+(D)$ has the same analytic form as it would have if the photons were massive.

To explain these properties in more detail, let us first explain the recipe of [1] about how to separate the coordinate space Feynman function into its classical and residual quantum parts. To be specific, let us discuss the case where D is the following triangle graph.

D:



Let D' denote the following graph corresponding to an electro-magnetic correction:

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