

Regular Holonomic Systems and their Minimal Extensions II

Application to the Multiplicity Formula for Verma Modules

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§ 0. Introduction

(0.1) This note together with [10] is an introduction to a part of Professor Kashiwara's lectures at RIMS in 1981. As explained in [10], the contents of the lectures were concerned with the recent development of the regular holonomic systems and its application to the representation theory of a semisimple Lie algebra.

In this note, we give a report on the part of "its application to the representation theory".

(0.2) The multiplicity formula for Verma modules conjectured by Kazhdan-Lusztig [7] was proved by Brylinski-Kashiwara [3] and Beilinson-Bernstein [1].

In this note we give an outline of a proof of the multiplicity formula based on Kashiwara's lectures. Needless to say, we do not give a complete proof of all the results stated in [3]. We mainly give a summary and describe in detail the theorems whose proofs are slightly different from the ones in [3]. They are Theorems (3.3) and (3.4) in this note. We use the Beilinson-Bernstein Theorem in [1] to prove Theorem (3.3) and simplify the proof of Theorem (3.4) by using the Bernstein-Gelfand-Gelfand Theorem stated in (1.11.1).

(0.3) We frequently use the notation and results in [10] without notice. Accordingly we recommend the reader to consult the report [10] in reading this note. On the other hand, the report written by Tanisaki [12] contains topics related to the text of this note and further applications of the \mathcal{D} -Modules to the representation theory.

§ 1. The category $\tilde{\mathcal{O}}$

(1,1) Let \mathfrak{g} be a semisimple Lie algebra over \mathbb{C} and let \mathfrak{t} be a Cartan