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F-mild Hyperfunctions and Fuchsian Partial Differential Equations

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§ 0. Introduction

Non-characteristic boundary value problems were formulated for hyperfunctions by Komatsu-Kawai [9] and Schapira [12]. They defined the boundary values of hyperfunction solutions and proved the uniqueness of solutions of the boundary value problem. Solvability of the (local) boundary value problem was proved by Kaneko [2] under the assumption of semi-hyperbolicity.

Kataoka [6, 8] introduced the notion of mildness on the boundary for hyperfunctions. He studied non-characteristic boundary value problems in detail by using the theory of mild hyperfunctions (see [7, 8]).

Let P be a linear partial differential operator of order m with analytic coefficients defined on an open subset M of $\boldsymbol{R}^n \ni x=(x_1, x')$, and set $\operatorname{int} M_+=\{x \in M; x_1>0\}$ and $N=\{x \in M; x_1=0\}$. Suppose that N is non-characteristic with respect to P. Then any hyperfunction $u(x)$ defined on $\operatorname{int} M_+$ satisfying $Pu(x)=0$ becomes mild on N, and the boundary value $v_j(x')=(\partial/\partial x_1)^j u(+0, x')$ is defined as a hyperfunction on N for any integer $j \geqq 0$. Moreover if $v_0(x'), \cdots, v_{m-1}(x')$ vanish, then $u(x)$ vanishes near N.

However, if N is characteristic with respect to P, then $u(x)$ is not mild in general. In this paper, we define the F-mildness for hyperfunctions defined on $\operatorname{int} M_+$. The notion of F-mildness is a generalization of that of mildness. If $u(x)$ is F-mild on N, we can define the boundary value $v_j(x')=(\partial/\partial x_1)^j u(+0, x')$ for any integer $j \geqq 0$ as a hyperfunction on N in a natural way.

Using F-mild hyperfunctions, we formulate boundary value problems for Fuchsian partial differential operators and prove the uniqueness of solutions of the boundary value problem. Let P be a Fuchsian partial differential operator of weight $m-k$ with respect to x_1 in the sense of Baouendi-Goulaouic [1] and let $u(x)$ be a hyperfunction on $\operatorname{int} M_+$ satisfying $Pu(x)=0$. Assume that the characteristic exponents of P avoid certain integral values. Under these assumptions, if $u(x)$ is F-mild on N

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