Advanced Studies in Pure Mathematics 4, 1984
Group Representations and Systems of Differential Equations
pp. 209–221

Regular Holonomic Systems and their Minimal Extensions I

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This note, together with J. Sekiguchi [13], is intended to be an introduction to Professor Kashiwara's lectures at RIMS in 1981. At that time, he lectured on three topics as follows:

(i) *Gabber's theorem on the involutiveness of the characteristic variety of a coherent $\mathscr{D}_X$-Module.*

(ii) *Some fundamental results on regular holonomic systems (holonomic systems with regular singularities).*

(iii) *An application of regular holonomic systems to the representation theory of a semisimple Lie algebra.*

Based on his lectures, we will make here a survey of (i) and (ii) above. As to (iii), the reader is referred to J. Sekiguchi [13].

Throughout this note, X stands for a complex manifold. We denote by T^*X the cotangent bundle of X with canonical projection $\pi\colon T^*X \to X$. If Y is a submanifold of X, the conormal bundle of Y in X will be denoted by T_Y^*X. We also use the notations $\mathring{T}^*X = T^*X \setminus T_X^*X$ and $\mathring{\pi} = \pi|_{\mathring{T}^*X}$. As usual, we denote by $\mathscr{D}_X$ the Ring over X of linear differential operators of finite order and by $\mathscr{E}_X$ the Ring over T^*X of microdifferential operators of finite order, respectively. In Section 2 and Section 3, we will freely use the terminology of derived categories. For a Ring $\mathscr{A}$ on X, we denote by $\mathrm{D}(\mathscr{A})$ the derived category of the category of (left) $\mathscr{A}$-Modules.

§ 1. Regular holonomic systems

Let Ω be an open subset of $\mathring{T}^*X = T^*X \setminus T_X^*X$ and V a conic involutive closed analytic subset of Ω. Then we define $\mathscr{J}_V$ to be the sub-Module of $\mathscr{E}_X(1)|_\Omega$ consisting of all microdifferential operators P whose symbols $\sigma_1(P)$ vanish on V. We denote by $\mathscr{A}_V = \bigcup_{k\geqslant 1} \mathscr{J}_V^k$ the sub-Algebra of $\mathscr{E}_X|_\Omega$ generated by $\mathscr{J}_V$. Note that $\mathscr{J}_V$ is a bilaterally coherent $\mathscr{E}_X(0)|_\Omega$-Module.

Proposition 1.1. *For a coherent $\mathscr{E}_X|_\Omega$-Module M, the following conditions are equivalent:*

Received April 25, 1983.