Advanced Studies in Pure Mathematics 4, 1984
Group Representations and Systems of Differential Equations
pp. 181–208

Symbols and Formal Symbols of Pseudodifferential Operators

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Introduction

In this paper, we present a symbol theory of pseudodifferential operators in analytic category. A pseudodifferential operator is, by definition, an integral operator

$$u(x) \longmapsto Pu(x) = \int K(x, x')u(x')dx' \tag{0.1}$$

with a holomorphic microfunction kernel $K(x, x')$ defined on the conormal bundle supported by $x=x'$. The sheaf of rings of pseudodifferential operators is denoted by $\mathscr{E}^R$ ([5], [6], [8]). It follows from Cauchy's integral formula that $\mathscr{E}^R$ contains all linear differential operators with analytic coefficients. Moreover, $\mathscr{E}^R$ includes the sheaf $\mathscr{E}^\infty$ of microdifferential operators ([10]). Needless to say, those classes of operators are very important in the investigations of various problems. We emphasize that the classes contain operators of infinite order and that the use of such operators is crucial in many cases (cf. [6], [10], [12]).

Symbols of pseudodifferential operators are defined by Kataoka [7]. He defines symbols from the cohomological definition of $\mathscr{E}^R$ by the aid of Radon transformations. On the other hand, Boutet de Monvel [4] introduces analytic pseudodifferential operators by using oscillatory integrals for given symbol classes and shows that standard symbolic calculus is valid as well as in C^∞-category (see [11], for example). We note that pseudodifferential operators in the sense of [4] are contained in $\mathscr{E}^R$ by virtue of Kataoka's theory.

The aim of this paper is to develop and to complete the symbol theory of $\mathscr{E}^R$ from the standpoint of [7]. The advantages of the viewpoint are related to the invariance of the cohomological definition of $\mathscr{E}^R$. The sheaf itself is defined independently of a choice of local coordinate systems. The cohomology group which defines $\mathscr{E}^R$ can be represented elementarily by the method of the Radon transformation ([7], [8]); we shall make full use of the method. One of our main contributions is introducing the

Received March 31, 1983.