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On Riemannian Manifolds of Nonnegative Ricci Curvature Containing Compact Minimal Hypersurfaces

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§ 0. Introduction

In this paper we study geometric properties of Riemannian manifolds which contain compact minimal hypersurfaces. Our main result, Theorem 4.1, of this paper is stated as follows.

Let N be an n-dimensional $(n \geqslant 2)$ connected, complete, real analytic Riemannian manifold without boundary. Let (M_1, f_1) and (M_2, f_2) be compact, connected, real analytic, minimal hypersurfaces immersed in N by real analytic immersions f_1 and f_2. Suppose that N is of nonnegative Ricci curvature and that $f_1(M_1) \cap f_2(M_2) = \phi$. Then N is isometric to one of four types of real analytic Riemannian manifolds described in Section 4.

In case N is a complete, connected, locally symmetric space of nonnegative sectional curvature, such classification was already done by Nakagawa and Shiohama ([6]).

Theorem 1.1 in Section 1 plays important roles in this paper which was proved by the author ([4]). In Section 2 we give an application of Theorem 1.1. Making use of this theorem, we obtain Lemma 2.1 which is a basic lemma of this paper. In Section 3 we study geometric properties of compact, connected Riemannian manifolds with boundary which contain a compact minimal hypersurface. Results of this section will be used to prove Theorems 4.1 and 4.2. Theorem 3.1 was also proved by Kasue independently ([5]).

As applications of Theorem 4.2 we obtain Theorems 4.3 and 4.4. For connected, complete Riemannian manifolds of positive Ricci curvature, Frankel proved the assertions of Theorems 4.3 and 4.4 ([3]). But, in general, Frankel's result does not hold for Riemannian manifolds of nonnegative Ricci curvature. We can easily give counterexamples. Therefore, in our theorems, we need the assumption that Riemannian manifolds are homogeneous.

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