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Conformal and Killing Vector Fields on Complete Non-compact Riemannian Manifolds

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0. In this note, we introduce the notion of vector fields with finite global norms, in order to discuss the vector fields on non-compact Riemannian manifolds. It should seem to be natural notion because we have some generalizations of well-known results for compact Riemannian manifolds (cf. [3], [9]). These generalizations are our main results. Our discussions are restricted to conformal and Killing vector fields. We show some examples in which the relations between the volumes of complete non-compact Riemannian manifolds and the global norms of Killing vector fields are discussed. For Killing vector fields with finite global norms, the case of complete non-compact Riemannian manifolds without boundary has stated in [11], and the case of non-compact Riemannian manifolds with boundary has stated in [12]. Our idea is based on in [1], [4], [6] and [10]. The case of affine and projective vector fields with finite global norms may be discussed similarly, but this case is not stated in this note (cf. [13]).

The discussions of different point of views appeared in [5] and [7].

We shall be in C^∞-category. The manifolds considered are connected and orientable.

1. Let M be a complete non-compact Riemannian manifold (without boundary) of dimension m. We denote the Riemannian metric (resp. the Levi-Civita connection) on M by g (resp. ∇). Let g_{ij} denote the components of g with respect to a local coordinate system $(x^1, \cdots, x^m)$, and (g^{ij}) denotes the inverse matrix of the matrix (g_{ij}). We set $\nabla_i = \nabla_{\partial/\partial x^i}$ and $\nabla^i = g^{ij}\nabla_j$.

For two $(0, s)$-tensor fields T and S on M, we denote the local scalar product (resp. the global scalar product) of T and S by $\langle T, S\rangle$ (resp. $\langle\langle T, S\rangle\rangle$), that is,

$$\langle T, S\rangle = \frac{1}{s!} T_{i_1\cdots i_s} S^{i_1\cdots i_s}$$

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