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Hadamard Manifolds

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As the starting point in the study of Riemannian manifolds of nonpositive curvature, we first recall the Cartan-Hadamard theorem (cf. [9], [17], [68]).

Theorem. *Let H be an n-dimensional simply connected complete Riemannian manifold of nonpositive curvature. Then H is diffeomorphic to the n-dimensional Euclidean space $\boldsymbol{R}^n$. More precisely, at any point $p \in H$, the exponential mapping* $\exp_p$: $H_p \to H$ *is a diffeomorphism.*

This theorem presents a clear contrast to Meyer's theorem (cf. [9], [17], [68]): if a complete Riemannian manifold M is of strictly positive Ricci curvature, i.e., Ricci curvature $\geqq k > 0$ for some k, then M is compact.

A simply connected complete Riemannian manifold of nonpositive curvature is called a *Hadamard manifold* or a *Cartan-Hadamard manifold* after the Cartan-Hadamard theorem. Unless otherwise mentioned, H will always denote a Hadamard manifold throughout this report.

From the Cartan-Hadamard theorem, there follow several basic properties of Riemannian manifolds of nonpositive curvature. For example, any pair of distinct points of a *Hadamard manifold* can be joined uniquely by a geodesic segment. It also follows that the fundamental group of a compact Riemannian manifold of nonpositive curvature is an infinite group.

The primary object of this survey article is to investigate the behavior of geodesics of a Hadamard manifold. Then we apply these investigations to the study of the isometry groups, discrete subgroups of the isometry groups of Hadamard manifolds and the fundamental groups of compact Riemannian manifolds of nonpositive curvature.

The geodesic behavior in respect to the ergodicity of the geodesic flows on compact Riemannian manifolds of negative curvature has been investigated by many authors. For this subject, there is Sunada's report [86] in this proceeding.

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