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On Topological Blaschke Conjecture I

Cohomological Complex Projective Spaces

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By a Blaschke manifold, we mean a Riemannian manifold (M, g) such that, for any point $m \in M$, the tangential cut locus C_m of m in T_mM is isometric to the sphere of constant radius. There are some equivalent definitions (see Besse [2, 5.43]). The Blaschke conjecture is that any Blaschke manifold is isometric to a compact rank one symmetric space. If the integral cohomology ring of M is equal to the sphere S^k, or the real projective space RP^k, this conjecture is proved by Berger with other mathematicians [2, Appendix D]). We consider the case where the cohomology ring of M is equal to that of the complex projective space CP^k.

We obtain the following theorem.

Theorem. *Let (M, g) be a $2k$-dimensional Blaschke manifold such that the integral cohomology ring is equal to that of CP^k. Then M is PL-homeomorphic to CP^k for any k.*

Blaschke manifolds with other cohomology rings will be treated in subsequent papers.

If (M, g) is a Blaschke manifold and $m \in M$, Allamigeon [1] has shown that the cut locus $C(m)$ of m in M is the base manifold of a fibration of the tangential cut locus C_m by great spheres. We study the base manifold of such fibration by great circles. We apply the Browder-Novikov-Sullivan's theory in the classification of homotopy equivalent manifolds (see Wall [4]). Calculation of normal invariants gives our theorem. In Appendix, we give examples of non-trivial fibrations of S^3 by great circles. The author thanks to T. Mizutani and K. Masuda for the discussion of results in Appendix.

Detailed proof will appear elsewhere.

§ 1. Projectable bundles

In the paper [3], we have obtained a method of a calculation of the

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