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On a Minimal Helical Immersion into a Unit Sphere

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§ 0. Introduction

Let $f: M \to \overline{M}$ be an isometric immersion of a Riemannian manifold M into a Riemannian manifold $\overline{M}$. If the image curve of each geodesic of M has universal constant curvatures and its osculating order is d, then f is said to be a helical immersion of order d. In the present paper we shall study minimal helical immersions into a unit sphere.

A compact Riemannian manifold admits a minimal helical immersion into a unit sphere if and only if it is a strongly harmonic manifold (see [3], [6]). Therefore the classification of compact Riemannian manifolds which admit a minimal helical immersion into a unit sphere is equivalent with that of strongly harmonic manifolds. So taking account of [4], [6], [7] and [8], we may conjecture that if the order d of a minimal helical immersion $f: M \to S(1)$ into a unit sphere $S(1)$ is odd, then M is isometric to a sphere of constant curvature, if the order is even, then M is isometric to a compact rank one symmetric space and f is equivalent to a standard minimal immersion (for the definition, see [8]). To answer this conjecture, we must pay attension to the fact that M is a Blaschke manifold (cf. [1], [6]). In fact, if a Riemannian manifold is diffeomorphic to a sphere or real projective space and has a Blaschke structure, then it is isometric to one of the above spaces with canonical Riemannian structure (cf. [3]). On the other hand, in [6] we obtained an explicit expression of the helical immersion $f: M \to S(1)$ by a geodesic polar coordinate around a fixed point x in M, which contains the second fundamental form and their higher order covariant derivatives at x. Thus we can compute Jacobi fields along a geodesic issuing from x. Therefore we are interested in the relation between the Blaschke structure of M and the second fundamental form of f. The study of this relation is the main purpose of this paper.

Well we give the organization of this paper. We in Section 1 shall explain the results obtained in [6] and moreover study the induced metric

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