Advanced Studies in Pure Mathematics 3, 1984
Geometry of Geodesics and Related Topics
pp. 93–95

# On Deformations of the $C_l$-Metrics on Spheres

**Kazuyoshi Kiyohara**

**1.** This note is a summary of our recent result concerning the existence problem of deformations of the standard metric by $C_{2\pi}$-metrics on the $n$-dimensional sphere $S^n$. By definition a riemannian metric $g$ on a manifold $M$ is called a $C_l$-metric if all of its geodesics are closed and have the common length $l$. Let $\{g_t\}$ be a one-parameter family of $C_{2\pi}$-metrics on $S^n$ with $g_0$ being the standard one, and put

$$\frac{d}{dt} g_t|_{t=0} = h.$$

We call such a symmetric 2-form $h$ an infinitesimal deformation. It is known that each infinitesimal deformation $h$ satisfies the so-called zero-energy condition, i.e.,

$$\int_0^{2\pi} h(\dot{\gamma}(s), \dot{\gamma}(s)) ds = 0$$

for any geodesic $\gamma(s)$ of $(S^n, g_0)$ parametrized by arc-length (cf. [1] p. 151). In [3] we gave another necessary condition, the second order condition, for a symmetric 2-form to be an infinitesimal deformation, and showed that there are symmetric 2-forms which satisfy the zero-energy condition, but not satisfy the second order condition in the case of $S^n$ ($n \geq 3$). The present theorem is an extension of the result in [3]. We first review the second order condition, and then state the theorem.

**2.** Let $K_2$ be the vector space of symmetric 2-forms on $S^n$ which satisfy the zero-energy condition. Let $\sharp$ be the bundle isomorphism from the cotangent bundle $T^*S^n$ to the tangent bundle $TS^n$ obtained by the riemannian metric $g_0$. Define the function $\hat{h}$ on $T^*S^n$ for a symmetric 2-form $h$ by

$$\hat{h}(\lambda) = h(\sharp(\lambda), \sharp(\lambda)), \ \lambda \in T^*S^n.$$

Let $S^*S^n$ be the unit cotangent bundle with respect to the metric $g_0$. We

Received December 28, 1982.