

Notes on the Kolmogorov's Remark Concerning Classical Dynamical Systems on Closed Surfaces

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§ 1. Introduction

It is well-known that the geodesic flow on a closed surface with a negative curvature metric is ergodic. The geodesic flow can be regarded as the dynamical system describing the inertial motion of a particle on the surface with the metric. Since a closed surface with a negative curvature metric cannot be isometrically embedded in the three dimensional Euclidean space E^3 , this type of ergodic motion itself cannot be realized on a surface in E^3 . On the other hand, in the address of A.N. Kolmogorov [1] (see also Ya. G. Sinai [2]), he remarked that, around a closed surface of genus greater than one in E^3 , one can distribute a finite number of centers of attraction and repulsion such that the motion of a particle on M under these external forces is equivalent to the inertial motion in a negative curvature metric on M . As this remark was stated intuitively and without proof, one can only guess the details. The purpose of this note is to set up the situation in more detail and to prove a proposition which seems to be closely related to the remark.

In Section 2, we prepare some concepts in the theory of classical dynamical systems and state our results. In Section 3, the proposition in Section 2 is proved, where only standard methods are used. In Section 4, relations between our results and the Kolmogorov's remark are discussed.

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§ 2. Preliminaries and results

All manifolds and functions are assumed to be smooth in this paper. Let M be a manifold and TM (resp. T^*M) the tangent (resp. cotangent) bundle of M . Let $x=(x^i)$ be local coordinates on M and $(x, p)=(x^i, p_i)$