Advanced Studies in Pure Mathematics 3, 1984
Geometry of Geodesics and Related Topics
pp. 47–85

Geodesic Flows and Geodesic Random Walks

Toshikazu Sunada

Contents

§ 0. Introduction

A flow on a set X is a family of bijections $\varphi_t\colon X\to X$, $t\in \boldsymbol{R}$, which obeys group property $\varphi_{t+s}=\varphi_t\circ\varphi_s$. Such an object (X, φ_t) arises in many contexts of mathematics. A typical example is the shift operation on a mapping space $\mathrm{Map}(\boldsymbol{R}, M)$ defined by $(\varphi_t c)(s)=c(t+s)$. Although it may seem that this flow has no interesting feature at first sight, various examples of flows in differential geometry appear in fact as subshifts of $(\mathrm{Map}(\boldsymbol{R}, M), \varphi_t)$. For instance, let M be a connected complete Riemannian manifold, and let X be the set of all geodesics $c\colon \boldsymbol{R}\to M$. Then the

Received January 4, 1983.
Revised February 25, 1983.