

## Geodesic Flows and Geodesic Random Walks

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### § 0. Introduction

A flow on a set  $X$  is a family of bijections  $\varphi_t: X \rightarrow X$ ,  $t \in \mathbf{R}$ , which obeys group property  $\varphi_{t+s} = \varphi_t \circ \varphi_s$ . Such an object  $(X, \varphi_t)$  arises in many contexts of mathematics. A typical example is the shift operation on a mapping space  $\text{Map}(\mathbf{R}, M)$  defined by  $(\varphi_t c)(s) = c(t+s)$ . Although it may seem that this flow has no interesting feature at first sight, various examples of flows in differential geometry appear in fact as subshifts of  $(\text{Map}(\mathbf{R}, M), \varphi_t)$ . For instance, let  $M$  be a connected complete Riemannian manifold, and let  $X$  be the set of all geodesics  $c: \mathbf{R} \rightarrow M$ . Then the