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pp. 1–28

# On the Number of Closed Geodesics and Isomertry-Invariant Geodesics

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## Contents



## Introduction

This article is a survey of the Gromoll-Meyer theorem [13] on the number of closed geodesics and some theorems related to their theorem. The following is the theorem Gromoll and Meyer proved in 1969.

**Theorem.** *Any compact* 1*-connected riemannian manifold $M$ has infinitely many closed geodesics if the sequence of the Betti numbers for the free loop space of $M$ is unbounded.*

Though there is a long and rich history on closed geodesics on a compact riemannian manifold since Poincaré [38], Lusternik and Schnirelmann [30], etc., our survey covers an only small portion of the history. However the author believes that it is worth while introducing their method of proving the Gromoll-Meyer theorem and how the theorem has given influence to some theorems of closed geodesics. Note that no symmetric spaces of rank one satisfy the hypothesis on the Betti numbers. But there are many manifolds satisfying the assumption. Note also that the assumption is a topological one. It would be interesting to estimate the number of closed geodesics on a compact riemannian manifold, the quantity of differential geometry in terms of topological properties of manifolds only. From this point of view, it should be referred that Lusternik and Schnirelmann proved in 1929 that there exist at least three

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