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Residue Homomorphisms in Milnor K-theory

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In this paper, we give a generalization of the residue homomorphism by using Milnor's K-group [9], and study its relation with class field theory. Our residue homomorphism provides a very plain definition of the p-primary part of the reciprocity map in the local class field theory in characteristic $p>0$. This definition was used in Brylinski [4] for the study of ramifications in abelian extensions of local fields of characteristic $p>0$ and those of surfaces over finite fields. Our residue homomorphism also provides a description of the relation between the class field theory of a higher local field and that of its constant field (Section 4 Theorem 2).

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§ 0. Notations and preliminaries

Here we fix our notations and review some properties of Milnor's K-group. For a ring R, let $R^\times$ be the multiplicative group of all invertible elements of R. For a field k, let $K_q(k)$ $(q\geqq 0)$ be Milnor's K-group of k defined by generators $\{x_1, \cdots, x_q\}$ $(x_1, \cdots, x_q \in k^\times)$ and certain relations (cf. [9]). For a discrete valuation field k, let ord_k be the normalized additive discrete valuation of k. Let

$$O_k=\{x \in k;\ \mathrm{ord}_k(x)\geqq 0\}, \qquad m_k=\{x \in k;\ \mathrm{ord}_k(x)\geqq 1\},$$
$$U_k=\{x \in k;\ \mathrm{ord}_k(x)=0\}.$$

The residue class field of k is denoted by $\bar{k}$. For $x \in O_k$, let $\bar{x}$ be the residue class of x in $\bar{k}$. Concerning the Milnor K-group of a discrete valuation field k, for $i\geqq 1$, let $\bigoplus_{q\geqq 0} U^iK_q(k)$ be the graded ideal of $\bigoplus_{q\geqq 0}K_q(k)$ generated by elements a of $k^\times=K_1(k)$ such that $\mathrm{ord}_k(a-1)\geqq i$. Let

$$\hat{K}_q(k)=\varprojlim_i K_q(k)/U^iK_q(k),$$

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