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On the Absolute Galois Groups of Local Fields II

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Introduction

Let p be an odd prime number, $\boldsymbol{Q}_p$ the p-adic number field, k a finite algebraic extension of $\boldsymbol{Q}_p$ and $\bar{k}$ the algebraic closure of k. In [3], A. V. Jakovlev describes the absolute Galois group $G(\bar{k}/k)$ of k of even degree by using generators and relations (cf. [2]). However, this description is very complicated and not explicit. In [7], H. Koch says that a simple description of $G(\bar{k}/k)$ in terms of generators and relations seems impossible. Recently, in [5], Jannsen and Wingberg give a simple description of the absolute Galois group of k of any degree by using generators and relations. The purpose of this part is to give an account of the result of Jannsen and Wingberg [5]. This part is the sequel of Miki [8]. Readers are advised to recall the definition of Demuškin formation in [8].

Notation and terminology

Throughout this paper, $\boldsymbol{Z}$ and $\hat{\boldsymbol{Z}}$ denote the rational integer ring and the inverse limit of all finite cyclic groups, respectively. For a prime number p, we denote by $\boldsymbol{Z}_p$ the p-adic integer ring and by $\boldsymbol{Q}_p$ the p-adic number field. $\boldsymbol{F}_p$ denotes the prime field $\boldsymbol{Z}/p\boldsymbol{Z}$. For a profinite group G, we denote by $\tilde{G}$ the maximal pro-p-factor group of G. For elements $x, y \in G$, we put $[x, y]=xyx^{-1}y^{-1}$ and $x^y=yxy^{-1}$. For closed subgroups H and S of G, we denote by $[H, S]$ the closed subgroup of G generated by $\{[x, y] \mid x \in H, y \in S\}$. We denote by G^{ab} the factor group $G/[G, G]$. If G is commutative, we denote by G^* the dual group of G, by $\mathrm{Tor}(G)$ the torsion part of G and by $G(p)$ the p-part of G. Let A and B be G-modules. We denote by $A \oplus B$ the direct sum of A and B. We denote by $H^n(G, A)$ the n-th cohomology group of G with coefficients in A. Let s be a natural number and $(\boldsymbol{Z}/p^s\boldsymbol{Z})^\times$ the multiplicative group of the factor ring $\boldsymbol{Z}/p^s\boldsymbol{Z}$. Let α be a continuous homomorphism of G into $(\boldsymbol{Z}/p^s\boldsymbol{Z})^\times$. For elements $x+p^s\boldsymbol{Z} \in \boldsymbol{Z}/p^s\boldsymbol{Z}$ and $\sigma \in G$, we define $(x+p^s\boldsymbol{Z})^\sigma=\alpha(\sigma)(x+p^s\boldsymbol{Z})$. By this definition, we can regard $\boldsymbol{Z}/p^s\boldsymbol{Z}$ as G-module. We denote by $\boldsymbol{Z}/p^s\boldsymbol{Z}(\alpha)$ this G-module. From now on, p denotes an odd prime number.

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