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On the Absolute Galois Groups of Local Fields I

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§ 1. Introduction

Let p be an odd prime number and let $\boldsymbol{Q}_p$ be the field of p-adic numbers. Let k be a finite algebraic extension of $\boldsymbol{Q}_p$ and let G_k denote the absolute Galois group of k, i.e., the Galois group $G(\bar{k}/k)$ of the algebraic closure $\bar{k}$ of k. Jakovlev [7] [8] describes G_k in terms of generators and relations when $n=[k:\boldsymbol{Q}_p]$ is even. Recently, Jannsen and Wingberg [10] succeeded in giving a simpler description of G_k in terms of generators and relations for any k, by using Demuškin formation (a group theoretical characterization of G_k) due to Koch [13]. The purpose of the present paper is to give a historical exposition of a way to the concept of Demuškin formation, as the preliminaries of Komatsu [14]. We shall emphasize a number theoretical process and omit the proofs of the purely group theoretical parts.

§ 2. Šafarevič's theorem (the case where $\zeta_1 \notin k$)

Put $n=[k:\boldsymbol{Q}_p]$. Let $k(p)$ be the maximal p-extension of k and put $G_k(p)=G(k(p)/k)$. Let ζ_i be a primitive p^i-th root of unity for $i\geqq 1$. Let $L(i)$ be a free group of rank i and let $F(i)$ be a free pro-p-group of rank i, i.e., $F(i)=\varprojlim L(i)/N$, where the projective limit is taken over all normal subgroups N of $L(i)$ such that $L(i)/N$ are finite p-groups.

The following lemma is well known.

Lemma 1 (Schreier). *Any subgroup of $L(i)$ of index j is a free group of* rank $j(i-1)+1$.

By using Lemma 1 and local class field theory, Šafarevič [18] proves the following

Theorem 1. *Let the notation and assumptions be as above. Moreover, assume that $\zeta_1 \notin k$. Then $G_k(p)$ is a free pro-p-group of* rank $(n+1)$.

Proof. Put $G=G_0=G_k(p)$ and $G_{i+1}=[G_i, G_i]G_i^p$ for $i\geqq 0$, where

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