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On the Zeta Function of an Abelian Scheme over the Shimura Curve II*)

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Introduction

This paper is a continuation of our previous work [17] under the same title. Let F be a totally real number field of finite degree g over $\boldsymbol{Q}$, and B a division quaternion algebra over F which is unramified at one infinite prime of F, and ramified at all the other infinite primes of F. In [17], we developed a theory which generalizes that of Kuga and Shimura [11], for F and B as above assuming that $g=[F:\boldsymbol{Q}]$ is odd. Namely, when g is odd, we constructed an abelian scheme A_S over the Shimura curve V_S attached to B, and expressed the Hasse-Weil zeta function of A_S^k (the k-fold fibre product of A_S over V_S) as a product of Dedekind zeta functions and automorphic L-functions associated with $B^\times$. Also, as its application, we proved the Ramanujan-Petersson conjecture for certain automorphic forms on $B^\times$ for almost all finite primes of F.

The aim of this paper is to supplement [17] in the following two points:

(I) To obtain results parallel to that in [17] when g is even.

(II) To prove the Ramanujan-Petersson conjecture for all "good primes" of F.

The construction of A_S is carried out in Section 2. We will redo the construction in the case when g is odd also, for the sake of completeness. The main result of Section 2 is (2.6.2), which immediately enables us to extend the results of [17] for general F. To construct A_S, we use the functoriality of the canonical models, due to Deligne [3], which generalizes that of Shimura [22] Section 8 (cf. also [22] 2.13). We recall necessary tools for this in the first preliminary section. The main results of this paper are (3.1.4) and (3.2.1). The proof of (3.1.4) goes exactly in the same way as in [17] Sections 3–4 after (2.6.2), and we omit it, refering to [17] for details. (3.2.1) and its corollaries give an answer to the above (II).

We note that, as for the Ramanujan-Petersson conjecture, *Morita* [14] *has recently shown that the assertion* (3.2.2) *is valid without our assumption*

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