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Galois Groups and their Representations
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Notes on Metaplectic Automorphic Functions and Zeta Functions

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The purpose of the present notes it to state, without detailed proofs, several miscellaneous facts which may slightly help to imagine properties of zeros of various L-functions including Riemann's zeta function.

Basic facts in our arguments in the sequel are that the Möbius function is connected with Gauss sums through the relation (28), and that fairly precise properties of Gauss sums can be derived by means of certain generalized theta functions which may be regarded as automorphic functions on a metaplectic group, that is, an n-fold covering group of the adelized $SL(2)$ over an algebraic number field F. Here, n is a natural number with $n \geq 2$, but only odd n are really useful for our purpose.

In Section 1 and Section 2, we will investigate usual real analytic Eisenstein series on the upper half plane to explain the main way of thinking, and in Section 3 we will turn to the three dimensional upper half space to consider cases of actual meaning.

§ 1. Fourier coefficients of Eisenstein series

Throughout the present notes, $(a, b; c, d)$ will denote a 2×2 matrix with the first line a, b and the second line c, d, and an expression like $1+a/b$ or $a/b+1$ means exclusively $1+ab^{-1}$ or $ab^{-1}+1$, and never $(1+a)b^{-1}$ or $a(b+1)^{-1}$.

Let $H=\{z=x+iy \mid x \in \boldsymbol{R}, y>0\}$ be the upper half plane, and let Γ be a subgroup of $SL(2, \boldsymbol{R})$ acting on H discontinuously. We assume that Γ does not contain $(-1, 0; 0, -1)$, that $\Gamma \backslash H$ is of a finite volume, and that the stabilizer $\Gamma_\infty = \{\sigma \in \Gamma \mid \sigma\infty = \infty\}$ of ∞ in Γ contains the group of translations by $\boldsymbol{Z}$.

For a complex number s_0 with $\operatorname{Re} s_0 > 1$, a real analytic Eisenstein series $E(z, s_0)$, $(z \in H)$, is defined by

$$E(z, s_0) = \sum_{c,d} y^{s_0} |cz+d|^{-2s_0} \tag{1}$$

where the sum ranges over all pairs c, d such that there exists an element

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