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Weak Positivity and the Additivity of the Kodaira Dimension for Certain Fibre Spaces

Eckart Viehweg*)

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Let V and W be non-singular projective varieties over the field of complex numbers $\boldsymbol{C}$, $n=\dim(V)$ and $m=\dim(W)$. Let $f: V \to W$ be a fibre space (this simply means that f is surjective with connected general fibre $V_{\overline{W}}=V\times_W \operatorname{Spec}(\overline{\boldsymbol{C}(W)})$. We denote the canonical sheaves of V and W by ω_V and ω_W, and we write $\omega_{V/W}=\omega_V\otimes f^*\omega_W^{-1}$.

S. Iitaka conjectured the following inequality for the Kodaira dimension to be true:

Conjecture $C_{n,m}$. $\kappa(V)\geqslant\kappa(W)+\kappa(V_w)$.
Being more optimistic, one might ask:

Conjecture $C_{n,m}^+$. If $\kappa(W)\geqslant 0$ then

$$\kappa(V)\geqslant \operatorname{Max}\{\kappa(W)+\kappa(V_w), \operatorname{Var}(f)+\kappa(V_w)\}.$$

$\operatorname{Var}(f)$ is defined to be the minimal number k, such that there exists a subfield L of $\overline{\boldsymbol{C}(W)}$ of transcendental degree k over $\boldsymbol{C}$ and a variety F over L with $F\times_{\operatorname{Spec}(L)}\operatorname{Spec}(\overline{\boldsymbol{C}(W)})\sim V_w$ ($\sim$ means "birational").

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