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Weak Positivity and the Additivity of the Kodaira Dimension for Certain Fibre Spaces

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Let V and W be non-singular projective varieties over the field of complex numbers C, $n = \dim(V)$ and $m = \dim(W)$. Let $f: V \to W$ be a fibre space (this simply means that f is surjective with connected general fibre $V_W = V \times_W$ Spec $(\overline{C(W)})$. We denote the canonical sheaves of V and W by ω_V and ω_W , and we write $\omega_{V/W} = \omega_V \otimes f^* \omega_W^{-1}$.

S. Iitaka conjectured the following inequality for the Kodaira dimension to be true:

Conjecture $C_{n,m}$. $\kappa(V) \ge \kappa(W) + \kappa(V_w)$. Being more optimistic, one might ask:

Conjecture $C_{n,m}^+$. If $\kappa(W) \ge 0$ then

$$\kappa(V) \ge \operatorname{Max} \{\kappa(W) + \kappa(V_w), \operatorname{Var}(f) + \kappa(V_w)\}.$$

Var (f) is defined to be the minimal number k, such that there exists a subfield L of $\overline{C(W)}$ of transcendental degree k over C and a variety F over L with $F \times_{\text{Spec}(L)} \text{Spec}(\overline{C(W)}) \sim V_w$ (~ means "birational").

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