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Hodge Theory and Kodaira Dimension

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As explained in S. Iitaka's paper in this volume, "the addition conjecture" by Iitaka is now the central problem in the theory of Kodaira dimension. The theory of variation of Hodge structures by P. Griffiths, W. Schmid, P. Deligne and others provides a powerful tool for the study of this problem. In this paper we shall review such applications of Hodge theory. Section 1 is devoted to explaining the semi-positivity of the direct image sheaf of the relative canonical bundle, which is our fundamental result. In Section 2 we consider the direct image sheaf of higher multiples of the relative canonical bundle in the case where the base space is a curve. In Section 3 we shall discuss some global results by using the period domain. These results are completely covered by the papers [6], [7] and [8], which correspond to Sections 1, 2 and 3, respectively.

The semi-positivity theorems seem to have a close relationship with the existence of the moduli space of algebraic varieties. In this respect, we shall sketch in Section 2 an interesting application of our semi-positivity argument to deformation theory, due to K. Maehara.

All varieties and morphisms in this paper are defined over the complex number field.

For general concepts and results in the classification theory of algebraic varieties, we refer the reader to the papers of Iitaka and Viehweg in this volume, and the lecture notes by Ueno [16], which have become the "bible" in this field or Iitaka's textbook [20].

§ 1. Semi-positivity of the Hodge bundle

What we shall do is the *birational geometry* of algebraic varieties defined over the complex number field: any birationally equivalent varieties or morphisms are considered the same. Thus we usually have only to consider non-singular and projective varieties by Hironaka's resolution theorem. Also, a divisor which appears as the boundary of an open part of a variety will be simplified to a *divisor with normal crossings*, which is

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