

On the Structure of Compact Complex Manifolds in \mathcal{C}

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Notations and conventions.

We use the following convention and terminology (cf. Ueno [43]). A complex manifold is always assumed to be *connected*. A *complex variety* is a reduced and irreducible complex space. A *fiber space* is a proper surjective morphism of complex spaces with general fiber irreducible.

a) Let X be a compact complex manifold and L a line bundle on X . Then:

- $a(X)$: the algebraic dimension of X
- $q(X)$:= $\dim H^1(X, \mathcal{O}_X)$, the irregularity of X
- K_X : the canonical bundle of X
- $\kappa(L, X)$: the L -dimension of X
- $\kappa(X)$:= $\kappa(K_X, X)$ the Kodaira dimension of X

b) Let X be a compact complex variety, \tilde{X} a nonsingular model of X , $A \subseteq X$ an analytic subspace, E a holomorphic vector bundle on X . Then:

- $a(X)$ = $a(\tilde{X})$
- $q(X)$ = $q(\tilde{X})$
- $\text{Aut } X$: the complex Lie group of biholomorphic automorphisms of X
- $\text{Aut}_0 X$: the identity component of $\text{Aut } X$
- $\text{Aut}(X, A)$:= $\{g \in \text{Aut } X; g(A) = A\}$
- $\text{Aut}_0(X, A)$: the identity component of $\text{Aut}(X, A)$
- $P(E)$:= $(E - \{0\})/C^*$ ($\{0\}$ = the zero section of E)
- \mathcal{O}_X : the sheaf of germs of holomorphic vector fields on X

A compact complex variety X' is called a bimeromorphic model of X if it is bimeromorphic to X .

- c) Let Y be a complex variety. Then
- $\pi_1(Y)$: the fundamental group of Y with respect to some reference point.
- $\Omega(Y)$: the class of subsets of Y which is a complement of an at most countable union of proper analytic subvarieties of X , or