Advanced Studies in Pure Mathematics 1, 1983
Algebraic Varieties and Analytic Varieties
pp. 181–193

Higher Residues Associated with an Isolated Hypersurface Singularity

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§ 0. Introduction

The aim of this note is, as a preliminary to Prof. Saito's article in this volume, to give a brief introduction to the theory of primitive integrals associated with isolated hypersurface singularities which he is now developing extensively. The whole theory, though still incomplete, can be found in [8] and a summary in [7].

The central idea of this theory is an intimate connection between geometry and transcendental functions with the theory of elliptic integrals as a motivated example. In his case the geometric object is a (so far) local isolated hypersurface singularity, with which he associates a specific differential form called the primitive form. This primitive form enables us to study the singularity with analytic methods. Unfortunately, the existence of the primitive form has not yet been established in general except for a small but significant number of cases from explicit computations ([6]). The integrals of this form would give a new type of transcendental functions generalizing elliptic integrals of the first kind.

In order to state the fundamental properties of primitive forms which relate to geometric properties of singularity, we need to introduce what we call higher residue pairings (cf. [7]), whose definition is the goal of this note. Among many notions introduced by Saito with higher residues the exponent is one of the most important and is discussed in his article [9].

§ 1. A Hamiltonian system of an isolated hypersurface singularity

In this section we introduce what we call a Hamiltonian system which is the object of our study. In fact we treat a special kind of a Hamiltonian system: one which is associated with a universal unfolding of an isolated hypersurface singularity.

Definition (1.1). A *Hamiltonian system* $(X \xrightarrow{\phi} S \xrightarrow{\pi} T, \delta_1)$ is a collection of the following data: i) X, S, T are the germs of manifolds of

Received November 11, 1981.
Revised January 30, 1982.