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What is known about the Hodge Conjecture?

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In this talk we review the present state of our knowledge about the Hodge Conjecture, one of the central problems in complex algebraic geometry. In his 1950 Congress address [12], Hodge reported on the topological and differential-geometric methods in studying algebraic varieties and complex manifolds which had been initiated by Lefschetz and developped by Hodge himself. He raised there many problems, and most of them were settled in 1950's by extensive works due to Kodaira and others. One notable exception to this is the so-called *Hodge Conjecture* which, if true, will give a characterization of cohomology classes of algebraic cycles on a nonsingular projective variety, generalizing the Lefschetz criterion for the case of divisors. This conjecture has an arithmetic flavour, as is common to most problems concerning algebraic cycles, which makes the problem interesting and difficult at the same time.

In § 1, we recall the formulation of Hodge's general conjecture—the original one due to Hodge and modified version due to Grothendieck [8]. In § 2, we review the various cases where this conjecture has been verified. Although the examples are still not so many, the reader might notice substantial increase of the known cases as compared to those listed in [8] (1969). In § 3, we mention further examples some of which are new. A few remarks will be given in § 4.

§ 1. The formulation of the problem

Let V denote a nonsingular projective variety over $\boldsymbol{C}$, and let d be an integer such that $0<d<n=\dim V$. We use the same letter V to denote the compact complex manifold attached to V. As is well known (cf. [3]), an irreducible subvariety W of V (of dimension $r=n-d$) defines a topological $2r$-cycle on V so that its homology class $h(W)\in H_{2r}(V,\boldsymbol{Z})$ and the dual cohomology class $c(W)\in H^{2d}(V,\boldsymbol{Z})$ are well defined. The cohomology class of an algebraic cycle $Z=\sum n_i W_i$ $(n_i\in\boldsymbol{Z})$ is defined by linearity: $c(Z)=\sum n_i c(W_i)$. Let

$$\mathscr{C}^d(V)_{\boldsymbol{Z}}\subset H^{2d}(V,\boldsymbol{Z})$$

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