Advanced Studies in Pure Mathematics 1, 1983
Algebraic Varieties and Analytic Varieties
pp. 49–54

Holonomic Systems of Linear Differential Equations with Regular Singularities and Related Topics in Topology

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The purpose of this talk is to give applications of the theory of holonomic systems of linear differential equations with regular singularities (we shall call them in this note regular holonomic systems, for short). Regular holonomic systems appear, besides purely analytic applications, as tools to connect topological objects with geometric or algebraic objects. This comes from the fact that Hilbert's twenty-first problem holds for regular holonomic systems. In this note, we take two topics as examples: "intersection homology groups" and "vanishing cycle cohomologies".

§ 1. Hilbert's 21-st problem

1.1 We employ the same notations as in § 5 [0]. Then, Hilbert's 21-st problem for regular holonomic systems is stated as follows. (This was announced in [7].)

Theorem 1. The functor $\mathcal{DR}_X = \boldsymbol{R}\mathcal{H}om_{\mathcal{D}_X}(\mathcal{O}_X, \ ): \ D^b(\mathcal{D}_X)_{hr} \to D^b(\boldsymbol{C}_X)_c$ is an equivalence of the categories.

1.2 We shall discuss briefly how to construct the inverse functor Ψ: $D^b(\boldsymbol{C}_X)_c \to D^b(\mathcal{D}_X)_{hr}$ of $\mathcal{DR}_X$. Let $X_{\boldsymbol{R}}$ denote the underlying real analytic manifold of X, and let $\mathcal{D}b^{(0,p)}$ denote the sheaf of $(0,p)$-forms with distribution (in the sense of L. Schwartz) coefficients. A sheaf $\mathcal{F}$ of $\boldsymbol{C}$-vector spaces on $X_{\boldsymbol{R}}$ is called *$\boldsymbol{R}$-constructible* if there exists a stratification of $X_{\boldsymbol{R}}$ by subanalytic (see [2]) strata on which $\mathcal{F}$ is locally constant. For an $\boldsymbol{R}$-constructible sheaf $\mathcal{F}$, we define the subsheaf $\mathcal{TH}(\mathcal{F})^{(0,p)}$ of $\mathcal{H}om_{\boldsymbol{C}_X}(\mathcal{F}, \mathcal{D}b^{(0,p)})$ as follows: for any open subset U of X, $\Gamma(U, \mathcal{TH}(\mathcal{F})^{(0,p)}) = \{\varphi; \mathcal{F}|_U \to \mathcal{D}b^{(0,p)}|_U$; for any relatively compact open subanalytic subset V of U and for any $s \in \Gamma(V, \mathcal{F})$, there exists $u \in \Gamma(U, \mathcal{D}b^{(0,p)})$ such that $\varphi(s) = u|_V\}$. Then, $\mathcal{TH}(\mathcal{F})^{(0,p)}$ is an *exact* contra-

Received September 22, 1981.