Ultimate remarks

We wanted to have a first course of weak convergence that focuses on $\mathbb{R}^k$, $k \geq 1$, i.e., in finite dimension.

In this specific case, we do not need the full theory as developed in Billingsley (1968), van der Vaart and Wellner (1996) , Loève (1997) , Pollard (1984), Gaenssler (1983), Bogachev (2007a), Bogachev (2007b), etc.

The two most theoretical and sophisticated parts of the current development of weak convergence on a metric space $(E,\ d)$ are:

(a) the characterization of compact sets in $(E,\ d)$ which allows to correctly handling the asymptotic tightness problems of sequences of measures. In the space of real-valued continuous functions for example, the Arzèla-Ascóli theorem is the main characterization of relatively compact sets;

(b) The Vápnik-Červonenski classes of sets or classes of functions, also needed, to overcome difficulties in addressing uniform or asymptotic tightness.

These two points, and some others, make the approach of the books of weak convergence of measures vague as appalling and repulsive to those who are not mathematicians oriented people. To overcome those aspects, we wished to present an introduction book on the topic which is useful and powerful and yet kept simple.

Fortunately, compact sets are easy to characterize in $\mathbb{R}^k$, $k \geq 1$ as bounded and closed sets. They re well-described by the Bolzano-Weierstrass theorem. As a result, tightness is handling by Helly-Bray's theorem.

These facts allowed us: