CHAPTER 4

Specific Tools for Weak Convergence in $\mathbb{R}$

This chapter focuses on tools which are specific to convergence of sequences of real random variables. For such random variables, we may use Renyi's representations through uniform or exponential random variables, especially for sequences of independent and identically distributed random variables. Such representations use the generalized inverse functions on which concentrates the first section. Besides, in relation with Section 6 and Theorem 4.1 in Chapter 2, working on weak convergence in the same probability space may become a computation matter. This chapter gives tools for such an orientation.

1. Generalized inverses of monotone functions

This theory is done for non-decreasing and right-continuous functions . It may be done for non-increasing and left-continuous functions .

Sometimes, left or right continuity is not required (see Fact (9) below).

Let F be a non-decreasing and right-continuous function from $\mathbb{R}$ to $\mathbb{R}$. Let us define the generalized inverse of F as:

$$F^{-1}(u) = \inf\{x \in \mathbb{R}, F(x) \geq u\}, u \in \mathbb{R}.$$

Because of the importance of this transformation for univariate extreme value theory , we are going to expose important facts of generalized inverses. Since we want them to be known by heart, we expose all of them before we provide their proofs.

A - List of most important properties of the generalized inverses .

Fact (1). For any $u \in \mathbb{R}$ and for any $t \in \mathbb{R}$,