CHAPTER 1

Review of Usual Weak Convergence Results in $\mathbb{R}^k$

1. Introduction

In this chapter, we will see that most of the readers, actually know a considerable number of weak convergence results, even if they did not use this concept. What has to be done, on top of this review, is to present these individual results in the frame of a unified theory in the most general setting. This is the target of this book which will be given in the subsequent chapters.

Here, we are going to recall classical convergence results that any student should have encountered from the first courses in probability theory or in Statistics.

We begin to set the general frame of weak convergence in $\mathbb{R}^k$, $k \geq 1$. We will admit the statements in the following section. We will be able to establish their validity in Chapter 2, in particular in Theorem 4 of that chapter.

2. Weak Convergence in $\mathbb{R}^k$

Let us remind that the probability law of any random vector $X : (\Omega, \mathcal{A}, \mathbb{P}) \mapsto \mathbb{R}^k$ is characterized by

(a) its cumulative distribution function (*cdf*) :

$$\mathbb{R}^k \ni x \hookrightarrow F_X(x) = \mathbb{P}(X \leq x),$$

(b) its characteristic function (here, i is the complex number such that $i^2 = -1$ with positive sinus, and $< .,. >$ stands for the classical product space on $\mathbb{R}^k$)

$$\mathbb{R}^k \ni u \hookrightarrow \Phi(u) = \mathbb{E}(\exp(i < u, X >)),$$