CHAPTER 7

Continuous Random Variables

Until now, we exclusively dealt with discrete random variables. Introducing probability theory in discrete space is a good pedagogical approach. Beyond this, we will learn in advanced courses that the general Theory of Measure and Integration and Probability Theory are based on the method of discretization. General formulas depending on measurable mappings and/or probability measure are extensions of the same formulas established for discrete applications and/or discrete probability measures. This means that the formulas stated in this textbook, beyond their usefulness in real problems, actually constitute the foundation of the Probability Theory.

In this chapter we will explain the notion of continuous random variables as a consequence of the study of the cumulative distribution function (*cdf*) properties. We will provide a list of a limited number of examples as an introduction to a general chapter on probability law s.

Especially, we will see how our special guest, the **normal** or **Gaussian** probability law, has been derived from the historical works of *de Moivre* (1732) and Laplace (1801) using elementary real calculus courses.

Let us begin by the following definition.

Definition. Let X be a *rrv*. The following function defined from $\mathbb{R}$ to $[0,1]$ by

$$F_X(x) = \mathbb{P}(X \leq x) \text{ for } x \in \mathbb{R},$$

is called the **cumulative distribution function** *cdf* of the random variable X.