CHAPTER 5

Real-valued Random Variable Parameters

Before we begin, we strongly stress that the mathematical expectation is defined only for a real-valued random variable. If a random variable is not a real-valued one, we may have mathematical expectations of real-valued functions of it.

Let us introduce the notion of mathematical expectation in an every day life example.

Example 1. Consider a class of 24 students. Suppose that 5 of them have the age of 19, 7 the age of 20 , 10 the age of 23, and 2 the age of 17. Let us denote by m the average age of the class, that is :

$$m = \frac{5 \times 19 \ + \ 7 \times 20 \ + \ 10 \times 23 \ + \ 2 \times 17}{24} = 20.79.$$

Let X be the random variable taking the distinct ages as values, that is $\mathcal{V}(X) = \{x_1, ..., x_4\}$ with $x_1 = 19$, $x_2 = 20$, $x_3 = 23$, $x_4 = 17$, with probability law :

$$\mathbb{P}(X = x_1) = \frac{5}{24}, \ \mathbb{P}(X = x_2) = \frac{7}{24}, \ \mathbb{P}(X = x_3) = \frac{10}{24}, \ \mathbb{P}(X = x_4) = \frac{2}{24}.$$

We may summarize this probability law in the following table :

k	19	20	23	17
$P(x = k)$	$\frac{5}{24}$	$\frac{7}{24}$	$\frac{10}{24}$	$\frac{2}{24}$

Set Ω as the class of these 24 students. The following graph

$$\begin{array}{rccc} X: & \Omega & \to & \mathbb{R} \\ & \omega & \hookrightarrow & X(\omega) = \text{age of the student } \omega. \end{array}$$