CHAPTER 4

Random Variables

We begin with definitions, examples and notations.

Definition 1. Let $(\Omega, \mathcal{P}(\Omega), \mathbb{P})$ be a probability space. A random variable on $(\Omega, \mathcal{P}(\Omega), \mathbb{P})$ is an application X from Ω to $\mathbb{R}^k$.

If $k = 1$, we say that X is a **real-valued random variable**, abbreviated in (*rrv*).

If $k = 2$, X is a **random couple** or a **bi-dimensional random variable**.

If the general case, X is a random variable of k dimensions, or a k**-dimensional random variable**, or simply a **random vector**.

If X takes a finite number of distinct values (points) in $\mathbb{R}^k$, X is said to be a **random variable with finite number of values**.

If X takes its values in a set $\mathcal{V}_X$ that can be written in an enumerable form :

$$\mathcal{V}_X = \{x_1, x_2, x_3, ...\},$$

the random variable is said to be a **discrete random variable**. So, any random variable with finite number of values is a discrete random variable.

Examples 1. Let us toss twice a die whose faces are numbered from 1 to 6. Let X be the addition of the two occuring numbers. Then X is a real-valued random defined on

$$\Omega = \{1, 2, ..., 6\}^2$$