CHAPTER 3

Conditional Probability and Independence

Suppose that we are tossing a die three times and considering the outcomes in the order of occurring. The set of all individual events Ω is the set of triplets (i, j, k), where i, j and k are, respectively the face that comes out in the first, in the second and in the third tossing, that is

$$\Omega = \{1, 2, ..., 6\}^3 = \{(i,\ j,\ k),\ 1 \leq i, \text{J},\ k \leq 6\}.$$

Denote by A the event : **the sum of the three numbers i, j and k is six (6)** and by B the event : **the number 1 appears in the first tossing**. We have

$$\begin{aligned} A &= \left\{(i,j,k) \in \{1,2,...,6\}^3,\ i+j+k=6\right\} \\ &= \{(1,1,4),(1,2,3),(1,3,2),(1,4,1),(2,1,3),(2,2,2),(2,3,1),(3,1,2),(3,2,1),(4,1,4)\} \end{aligned}$$

and

$$\begin{aligned} B &= \left\{(i,j,k) \in \{1,2,...,6\}^3,\ i=1\right\} \\ &= \{1\} \times \{1,2,...,6\} \times \{1,2,...,6\} \end{aligned}$$

Remark that $Card(A) = 10$ and $Card(B) = 1 \times 6 \times 6 = 36$.

Suppose that we have two observers named **Observer 1** and **Observer 2**.

Observer 1 tosses the die three times and gets the outcome. Suppose the event A occurred. **Observer 1** knows that A is realized.

Observer 2, who is somewhat far from **Observer 1**, does not know. But **Observer 1** let him know that the event A occurred.