CHAPTER 2

Introduction to Probability Measures

Assume that we have a perfect die whose six faces are numbered from 1 to 6. We want to toss it twice. Before we toss it, we know that the outcome will be a couple (i, j), where i is the number that will appear first and j the second.

We always keep in mind that, in probability theory, we will be trying to give answers about events that have not occurred yet. In the present example, the possible outcomes form the set

$$\Omega = \{1, 2, ..., 6\} \times \{1, 2, ..., 6\} = \{(1, 1), (1, 2), ..., (6, 6)\}.$$

Ω is called the *sample* space of the experiment or the *probability space*. Here, the size of the set Ω is finite and is exactly 36. Parts or subsets of Ω are called events. For example,

(1) {(3, 4)} is the event : *Face 3 comes out in the first tossing and Face 4 in the second*,

(2) A = $\{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)\}$ is the event : *1 comes out in the first tossing* .

Any element of Ω, as a singleton, is an elementary event. For instance $\{(1, 1)\}$ is the elementary event : *Face 1 appears in both tossing* .

In this example, we are going to use the perfectness of the die, and the regularity of the geometry of the die, to the conviction that

(1) All the elementary events have equal chances of occurring, that is one chance out of 36.