SEQUENTIAL CONFIDENCE INTERVALS WITH BETA PROTECTION IN ONE-PARAMETER FAMILIES*

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A sequential confidence interval (CI) for a real parameter γ of the form $[L,\infty)$ is proposed, based on a consistent and asymptotically normal sequence $T_1, T_2, \ldots$ of real valued statistics. This CI is required to satisfy the coverage probability $P_\gamma\{L \leq \gamma\} \geq 1 - \alpha$ for every γ, and to provide beta protection at $\phi(\gamma)$: $P_\gamma\{L \leq \phi(\gamma)\} \leq \beta$ for every γ, where α, β, and the function $\phi(\gamma) < \gamma$ are given. It is shown that this can be achieved (under certain regularity assumptions) with a stopping time of the form N = least integer $n \geq r + c^2\tau^2(T_n)$ and a terminal decision $L = \rho(T_N)$, in which the functions τ and ρ depend on ϕ and the asymptotic variance σ^2. Asymptotic values are derived for $P_\gamma\{L > \gamma\}$ and $P_\gamma\{L \leq \phi(\gamma)\}$ as γ varies over values for which $\tau(\gamma) \to \infty$.

1. Introduction.

Let $T_1, T_2, \ldots$ be a sequence of real valued random variables whose joint distribution P_γ depends on a parameter γ with values in an interval Γ. Suppose a one-sided confidence interval (CI) for γ is desired of the form $[L,\infty]$, in which $L = L(T_1,T_2,\ldots)$, that satisfies the two conditions

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