NEGATIVE DEPENDENCE

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1. Introduction

Various concepts of positive dependence have been considered in the literature. Many of these concepts have been developed to obtain conditions on a random vector $\underline{T} = (T_1,\ldots,T_n)$ such that

$$(1) \qquad P\{T_1 > t_1,\ldots,T_n > t_n\} \geq \prod_{i=1}^{n} P\{T_i > t_i\} \text{ for all } t_1,\ldots,t_n \ .$$

See Lehmann (1966) for a discussion of the bivariate case and Barlow and Proschan (1975) and Block and Ting (1981) for details in the multivariate case.

There are many distributions such as the multinomial and the Dirichlet for which the reverse inequality holds (see Mallows (1968) and Jogdeo and Patil (1975)). However no systematic study of negative dependence concepts in the $n \geq 3$ case was attempted until recently (the bivariate case was considered by Lehmann (1966)).

In this paper we discuss various multivariate concepts of negative dependence. Many of these concepts arose out of discussions which were begun at the NSF/CBMS conference held in Columbia, Missouri in June 1979 at which Frank