FOURIER INTEGRAL ESTIMATE OF THE FAILURE RATE FUNCTION AND ITS MEAN SQUARE ERROR PROPERTIES

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1. Introduction

The failure rate function h is important in reliability and biometry. Estimates of h using weighting functions or "kernels" are quite common in the literature (see Singpurwalla and Wong (1982b)). The kernels that have been considered so far are *nonnegative* and *absolutely integrable in* $(-\infty,\infty)$. (Kernels satisfying this latter condition are known as L^1 kernels.) Singpurwalla and Wong (1982a) -- abbreviated as SW (1982a) -- have shown that the mean square error (MSE) of a kernel estimator of h using a compact L^1 kernel restricted to be nonnegative has an optimal rate of convergence of at most $0(n^{-4/5})$, regardless of the smoothness of h; n is the sample size. If the nonnegativity condition of the compact L^1 kernel is relaxed, and if h is $(m+1)$ times continuously differentiable, then (for $m > 2$), the rate of convergence of the MSE (can be improved and) is at most $0(n^{-2m/(2m+1)})$. A method for producing kernel estimators having the above property is the generalized jackknife of Gray and Schucany (1972). Specifically, if we use the generalized jackknife on two kernel estimators of h, with each estimator being based upon a nonnegative com-