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INFERENCE FOR LINEAR MODELS WITH RADIALLY DECOMPOSABLE ERROR

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This paper considers a linear model with an error term following a so-called radially decomposable multivariate distribution which can be represented as an independent product of a random scalar component, called the radial component, and a vector component, called the base component. The radially decomposable distributions include many symmetric multivariate distributions such as spherically symmetric distributions, ℓ_1-norm symmetric distributions, Liouville multivariate distributions and α-symmetric distributions with $\alpha = 1$. It is shown that the inference for the regression parameter $\boldsymbol{\beta}$ depends only on the distribution of the base component. Some consequences of this fact are also discussed.

1. Introduction. In many applications of the following linear model

$$\boldsymbol{Y} = \boldsymbol{X\beta} + \boldsymbol{\epsilon},$$

the errors are known to be non-normal. If the errors are i.i.d., one may consider the non-normal univariate distributions such as the Cauchy, Student's t, Laplace's double exponential, logistic, extreme value distributions and the stable-law distribution. If the errors are identically distributed but not independent, multivariate versions of these and other distributions are usually adopted for the error vector. In general, inference for linear models having multivariate error distributions is a thorny problem. Some of these multivariate distributions can be represented as an independent product of a random scalar component and a random vector component. In this case, the multivariate distribution is said to be *radially decomposable*, the scalar variable being called the *radial component* and the vector component the *base component.* In section 3, we shall show that the inference for the regression parameters depends only on the distribution of the base component, but not on that of the radial component. A change in the distribution of the radial component will only affect the inference of the scale parameter of the error term.

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