

SOME APPLICATIONS OF MONOTONE TRANSFORMATIONS IN STATISTICS¹

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A number of results concerning monotone transformations of random variables are reviewed. Particular attention is paid to the effects of choice of monotone scaling in two settings: (a) describing and quantifying dependence between two random variables, and (b) comparing two populations with ordinal categorical responses.

Properties of the concordant and discordant monotone correlation coefficients (Kimeldorf, May and Sampson (1982)) between random variables X and Y are discussed, and computational approaches are considered.

The two sample problem is explored where responses are ordinal categories and typical statistical procedures involve the arbitrary choice of monotone scales. The effects of the choice of scaling upon the resultant analyses are examined in detail.

1. Introduction

In a variety of situations, it is of interest to consider how the results of the analyses change when we transform the relevant random variables by monotone functions. The usual purpose of this is to study the effects of monotonically rescaling the measured random quantities. Depending on our needs, we might want a statistical procedure that is invariant to monotone scale changes, or we might want to choose an appropriate scaling in situations where the natural choice of scales is not clear. The purpose of this paper is to review some results in this area focusing on a somewhat less than standard usage of monotone transformations.

Traditional concerns about monotone invariance can lead to various notions. In some settings it leads to considering statistical procedures which depend only on ranks of the data. For jointly distributed random variables, it can lead to a discussion of procedures which depend solely on the copula

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