A CONSTRUCTION FOR PROCESSES WITH HISTORY-DEPENDENT TRANSITION INTENSITIES

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Summary

At the Sheffield meeting in honour of Joe Gani I presented the material of Whittle (1990); this was essentially Theorem 1 below and applications of it in network contexts. In the ensuing discussion John Bather raised an interesting point, which I now pursue.

1. Introduction. Suppose that a continuous-time Markov process has state variable x and infinitesmal generator T. Its equilibrium density $\pi(x)$ relative to a suitable measure μ then satisfies $T'\pi = 0$, where T' is the operator adjoint to T in that $\int (\pi T f)\mu(dx) = \int (f T'\pi)\mu(dx)$ for functions $\pi(x)$, $f(x)$. Consider now a family of such processes parametrised by a vector $\nu = \{\nu_j\}$ for which the generator has the form

$$T(\nu) = \sum_{n} \nu_j T_j \tag{1}$$

Here the T_j are a set of fixed generators and ν_j a set of variable parameters, scalar and non-negative. One can regard T_j as the generator of transitions by a particular mode (the j^{th} mode); these different modes being weighted differently as we vary the parameters ν_j. We shall refer to the vector $\nu = \{\nu_j\}$ as the rate vector; its essential property is that it enters linearly into the generator.

Let the family of processes generated as ν varies in some given set N be denoted F. Let us also suppose that the equilibrium density $\pi(x|\nu)$ of the process with intensity (1) is unique and known, for all ν in N. Note that prescription (1) includes also the non-homgeneous case

$$T(\nu) = T_0 + \sum_{j \neq 0} \nu_j T_j \tag{2}$$

This corresponds to the case when $\nu_0 \equiv 1$ for ν in N.

Consider now the mixed density

$$\tilde{\pi}(x) = \int \pi(x|\nu)\, \phi(d\nu) \tag{3}$$