PREQUENTIAL DATA ANALYSIS

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Abstract

The basic theory of the prequential approach to data analysis is described, and illustrated by means of both simulation experiments and applications to real data-sets.

Introduction

The prequential approach to the problems of theoretical statistics was introduced by Dawid (1984). It is based on the idea that statistical methods should be assessed by means of the validity of the predictions that flow from them, and that such assessments can usefully be extracted from a sequence of realized data-values, by forming, at each intermediate time-point, a forecast for the next value, based on an analysis of earlier values. The main emphasis is on probability forecasting, requiring that one describe current uncertainty about the predictand by means of a fully specified probability distribution. However, point forecasts, or other forms of prediction, can also be accommodated.

The purpose of the above paper was to indicate the fertility of the prequential point of view for furthering understanding of traditional concerns of theoretical statistics, such as consistency and efficiency. However, the prequential approach is essentially data-analytic. As such, it is particularly well suited to empirical investigation of the structure and properties of real-world observations, and their sources. In this paper, we shall discuss some of the ways in which prequential assessment may be applied in practical problems, including goodness-of-fit, model choice and density estimation. These methods are illustrated, by means of simulation experiments and applications to real data.

Prequential Assessment

Let $\underset{\sim}{Y} = (Y_1, Y_2, \ldots)$ be a potentially infinite sequence of observables, and $\underset{\sim}{Y}^{(k)} = (Y_1, Y_2, \ldots, Y_k)$. We consider methods of forming, for each $k = 1, 2, \ldots$, a *prediction*, $\hat{y}_k$, for Y_k, based on past data $\underset{\sim}{Y}^{(k-1)} = \underset{\sim}{y}^{(k-1)}$; or, more generally, of deciding on an *action* a_k on the basis of $\underset{\sim}{y}^{(k-1)}$, when subject to a loss $L_k(y, a)$ if $Y_k = y$ and $a_k = a$. Such a method M having been applied for $k = 1$ to n, and resulting in actions $(a_1, a_2, \ldots, a_n)$, its performance might be assessed by means of its total *prequential loss*

$$L_n^*(M) = \sum_{k=1}^{n} L_k(y_k, a_k),$$

which measures the success of its earlier forecasts; and comparison amongst