# ON STOCHASTIC DEPENDENCE AND A CLASS OF DEGENERATE DISTRIBUTIONS

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We investigate the approximation of stochastic dependence by functional relationships involving so-called cyclic permutations of the interval.

**1. Introduction.** Dependence between two random variables can take, of course, a variety of forms, of which stochastic independence and functional dependence can be argued to be most opposite in character. In the one case, neither variable provides any information about the other, whereas in the second case there is complete determination (or *complete dependence*: Lancaster, 1963). By means of a direct construction for uniform variables, Kimeldorf and Sampson (1978) showed, however, that one can pass continuously from one to the other of these situations in the natural sense of weak convergence. This obviously weakens complete dependence as a foil for independence (and led Kimeldorf and Sampson, 1978, to the fruitful concept of monotone dependence). Indeed, couched in somewhat different language, Theorem 1 of Brown (1966) can be read to state that *any* form of dependence between uniform random variables can be approximated in the weak sense by functionally related random variables.

On the other hand, this raises the question, of theoretical and obvious computational interest, of the extent to which complete dependence can be used to approximate forms of stochastic dependence. We pursue this in several directions. First we show that functional dependence can be specified to a highly stylized class of invertible functions, the so-called cyclic permutations of the interval. Second, we show that it is possible to move from two to an arbitrary finite number of random variables. Finally we extend to arbitrary (continuous) marginals. Regarding the last point, we systematically take the viewpoint of fixing marginals and consider dependence within this constraint; for the narrower question of regression in this context, see Vitale and Pipkin (1976) and Vitale (1979). We make extensive use of the *uniform representation* of random variables (Kimeldorf and Sampson, 1975).

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